Sin, Cos, and Tan Revealed

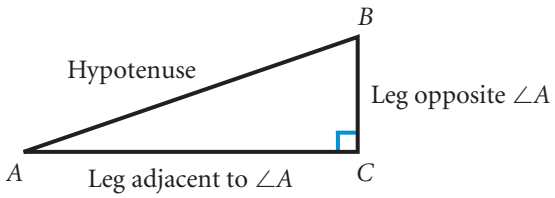
Reference

Did you ever wonder what those keys on your calculator that say "sin," "cos," and "tan" are all about? Well, here's where you find out.

You've seen that whenever two right triangles have another angle in common, the triangles must be similar, and so the corresponding ratios of lengths of sides in those triangles are equal.

These ratios depend only on that common acute angle. Each ratio of lengths in the right triangle has a name. The study and use of these ratios is part of a branch of mathematics called **trigonometry.**

Suppose you are given an acute angle—that is, an angle between 0 and 90 degrees. You can create a right triangle in which one of the acute angles is equal to that given angle. Suppose you label that triangle as shown in the diagram, so that $\angle A$ is equal to the acute angle you started with.



The trigonometric ratios are then defined as explained on the following pages. The principles of similarity guarantee that these ratios will be the same for every right triangle that has an acute angle the same size as $\angle A$.

Sine of an Angle

The **sine** of $\angle A$ is the ratio of the length of the leg opposite $\angle A$ to the length of the hypotenuse. The sine of $\angle A$ is abbreviated as sin A. For example, in $\triangle RST$, the leg opposite $\angle R$ has length 4, and the hypotenuse has length 7, so $\sin R = \frac{4}{7}$.

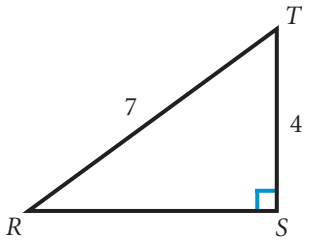
continued

In summary

$$\sin A = \frac{\text{length of leg opposite } \angle A}{\text{length of hypotenuse}}$$

Or simply,

$$\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$$



Cosine of an Angle

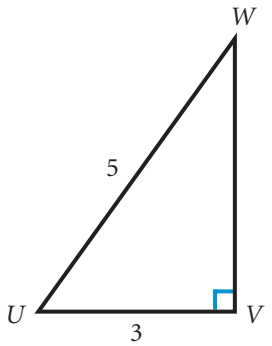
The **cosine** of $\angle A$ is the ratio of the length of the leg adjacent to $\angle A$ to the length of the hypotenuse. The cosine of $\angle A$ is abbreviated as $\cos A$. For example, in $\triangle UVW$, the leg adjacent to $\angle U$ has length 3, and the hypotenuse has length 5, so $\cos U = \frac{3}{5}$.

In summary

$$\cos A = \frac{\text{length of leg adjacent to } \angle A}{\text{length of hypotenuse}}$$

Or simply,

$$\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$$



Tangent of an Angle

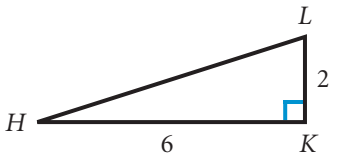
The **tangent** of $\angle A$ is the ratio of the length of the leg opposite $\angle A$ to the length of the leg adjacent to $\angle A$. The tangent of $\angle A$ is abbreviated as $\tan A$. For example, in $\triangle HKL$, the leg opposite $\angle H$ has length 2, and the leg adjacent to $\angle H$ has length 6, so $\tan H = \frac{2}{6}$.

In summary

$$\tan A = \frac{\text{length of leg opposite } \angle A}{\text{length of leg adjacent to } \angle A}$$

Or simply,

$$\tan A = \frac{\text{opposite}}{\text{adjacent}}$$



continued

Trigonometric Functions on a Calculator

Any scientific calculator or graphing calculator has keys that will give you the values of these functions for any angle.

In some calculators, you enter the size of the angle and then push the appropriate trigonometric ratio key. For other calculators, you do the opposite.

You have been measuring angles using degrees as the unit of measurement, but there are other units for measuring angles. Most calculators that work with trigonometric functions have a mode key that you can set to "deg."

The Tree and the Pendulum

1. Now that you have been introduced to trigonometry, it's time to look again at how to measure the height of a tree.

Here are the key facts.

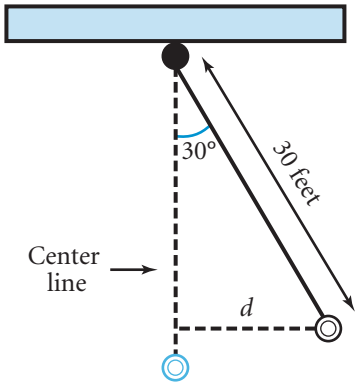
- Woody is 12 feet from the tree.
- Woody's line of sight to the top of the tree is at an angle of 70° up from horizontal.
- Woody's eyes are 5 feet off the ground.

Describe how Woody could find the height of the tree using trigonometry and these measurements.

2. You can apply trigonometry to the situation from the Algebra 1 unit *The Pit and the Pendulum*.

Suppose a 30-foot pendulum has an initial amplitude of 30°.

How far is the bob from the center line when the pendulum starts? In other words, what is the distance labeled d?

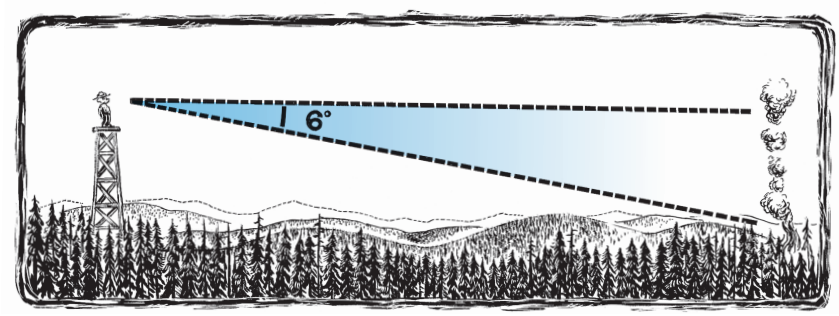


Sparky and the Dude

1. Sparky the Bear

Sparky the Bear is atop a 100-foot tower. He is looking out over a fairly level area for careless people who might start fires. Suddenly, he sees a fire starting. He marks down the direction of the fire. But he also needs to know how far away from the tower the fire is.

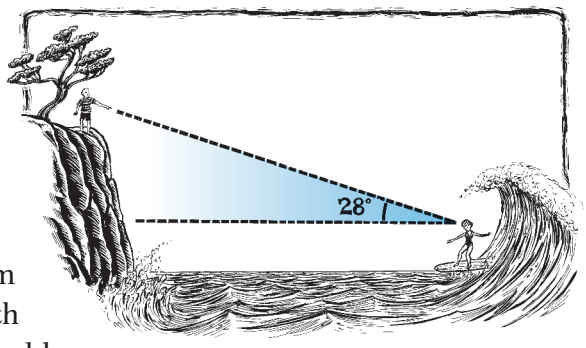
To figure out this distance, Sparky grabs his handy protractor. Because he is high up on top of the tower, he has to look slightly downward toward the fire. He finds that his line of sight to the fire is at an angle of 6° below horizontal. *Note:* This diagram is not to scale.



How far is the base of Sparky's tower from the fire?

2. Dude on a Cliff

Shredding Charlene is out surfing. She catches the eye of her friend Dave the Dude, who is standing at the top of a cliff. The angle formed by Charlene's line of sight and the horizontal measures 28°.



Charlene is 50 meters from the bottom of the cliff. Charlene and Dave are both 1.7 meters tall. They are both 16 years old. The surfboard is level with the base of the cliff.

How high is the cliff?