For each function that is one-to-one, write an equation for its inverse function. Give the domain and range of both f and f^{-1}. If the function is not one-to-one, say so.

15. $f(x) = 3x - 6$

16. $f(x) = 2(x + 1)^3$

17. $f(x) = 3x^2$

18. $f(x) = \dfrac{2x - 1}{5 - 3x}$

19. $f(x) = \sqrt[3]{5 - x^4}$

20. $f(x) = \sqrt{x^2 - 9}, \quad x \geq 3$

Write an equivalent statement in logarithmic form.

21. $\left(\dfrac{1}{10}\right)^{-3} = 1000$

22. $a^b = c$

23. $(\sqrt{3})^4 = 9$

24. $4^{-3/2} = \dfrac{1}{8}$

25. $2^x = 32$

26. $27^{4/3} = 81$

Solve each equation.

27. $3x = 7^{\log_7 6}$

28. $x = \log_{10} 0.001$

29. $x = \log_6 \dfrac{1}{216}$

30. $\log_x 5 = \dfrac{1}{2}$

31. $\log_{10} 0.01 = x$

32. $\log_x 3 = -1$

33. $\log_x 1 = 0$

34. $x = \log_2 \sqrt{8}$

35. $\log_x \sqrt[3]{5} = \dfrac{1}{3}$

36. $\log_{1/3} x = -5$

37. $\log_{10}(\log_2 2^{10}) = x$

38. $x = \log_{4/5} \dfrac{25}{16}$

39. $2x - 1 = \log_6 6^x$

40. $x = \sqrt{\log_{1/2} \dfrac{1}{16}}$

41. $2^x = \log_2 16$

42. $\log_3 x = -2$

43. $\left(\dfrac{1}{3}\right)^{x+1} = 9^x$

44. $5^{2x-6} = 25^{x-3}$

4.4 Evaluating Logarithms and the Change-of-Base Theorem

- Common Logarithms
- Applications and Models with Common Logarithms
- Natural Logarithms
- Applications and Models with Natural Logarithms
- Logarithms with Other Bases

Common Logarithms Two of the most important bases for logarithms are 10 and *e*. Base 10 logarithms are called **common logarithms.** The common logarithm of *x* is written $\log x$, where the base is understood to be 10.

Common Logarithm

For all positive numbers *x*,

$$\log x = \log_{10} x.$$

A calculator with a log key can be used to find the base 10 logarithm of any positive number.

EXAMPLE 1 Evaluating Common Logarithms with a Calculator

Use a calculator to find the values of

$$\log 1000, \quad \log 142, \quad \text{and} \quad \log 0.005832.$$

SOLUTION **Figure 33** shows that the exact value of $\log 1000$ is 3 (because $10^3 = 1000$), and that

$$\log 142 \approx 2.152288344$$

and $$\log 0.005832 \approx -2.234182485.$$

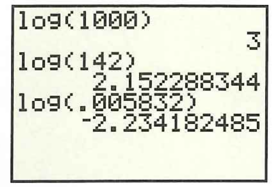


Figure 33

Most common logarithms that appear in calculations are approximations, as seen in the second and third displays.

✔ *Now Try Exercises 11, 15, and 17.*

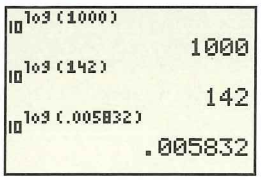


Figure 34

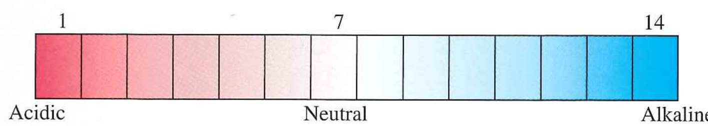
Figure 34 reinforces the concept presented in the previous section: **log x *is the exponent to which* 10 *must be raised to obtain x.***

NOTE ***Base a logarithms of numbers between* 0 *and* 1, *where* $a > 1$, *are always negative,*** as suggested by the graphs in **Section 4.3.**

Applications and Models with Common Logarithms In chemistry, the **pH** of a solution is defined as

$$\mathbf{pH = -\log[H_3O^+]},$$

where $[H_3O^+]$ is the hydronium ion concentration in moles* per liter. The pH value is a measure of the acidity or alkalinity of a solution. Pure water has pH 7.0, substances with pH values greater than 7.0 are alkaline, and substances with pH values less than 7.0 are acidic. It is customary to round pH values to the nearest tenth.



EXAMPLE 2 Finding pH

(a) Find the pH of a solution with $[H_3O^+] = 2.5 \times 10^{-4}$.

(b) Find the hydronium ion concentration of a solution with $\text{pH} = 7.1$.

SOLUTION

(a) $\text{pH} = -\log[H_3O^+]$

$= -\log(2.5 \times 10^{-4})$ — Substitute.

$= -(\log 2.5 + \log 10^{-4})$ — Product property (Section 4.3)

$= -(0.3979 - 4)$ — $\log 10^{-4} = -4$ (Section 4.3)

$= -0.3979 + 4$ — Distributive property (Section R.2)

$\text{pH} \approx 3.6$ — Add.

*A *mole* is the amount of a substance that contains the same number of molecules as the number of atoms in exactly 12 grams of carbon-12.

(b)

$$\text{pH} = -\log[\text{H}_3\text{O}^+]$$

$$7.1 = -\log[\text{H}_3\text{O}^+] \quad \text{Substitute.}$$

$$-7.1 = \log[\text{H}_3\text{O}^+] \quad \text{Multiply by } -1. \text{ (Section 1.1)}$$

$$[\text{H}_3\text{O}^+] = 10^{-7.1} \quad \text{Write in exponential form. (Section 4.3)}$$

$$[\text{H}_3\text{O}^+] \approx 7.9 \times 10^{-8} \quad \text{Evaluate } 10^{-7.1} \text{ with a calculator.}$$

✔ ***Now Try Exercises 29 and 33.***

NOTE In the fourth line of the solution in **Example 2(a),** we use the equality symbol, =, rather than the approximate equality symbol, ≈, when replacing log 2.5 with 0.3979. This is often done for convenience, despite the fact that most logarithms used in applications are indeed approximations.

EXAMPLE 3 Using pH in an Application

Wetlands are classified as *bogs, fens, marshes,* and *swamps* based on pH values. A pH value between 6.0 and 7.5 indicates that the wetland is a "rich fen." When the pH is between 3.0 and 6.0, it is a "poor fen," and if the pH falls to 3.0 or less, the wetland is a "bog." (*Source:* R. Mohlenbrock, "Summerby Swamp, Michigan," *Natural History*.)

Suppose that the hydronium ion concentration of a sample of water from a wetland is 6.3×10^{-5}. How would this wetland be classified?

SOLUTION

$$\begin{aligned}
\text{pH} &= -\log[\text{H}_3\text{O}^+] && \text{Definition of pH}\\
&= -\log(6.3 \times 10^{-5}) && \text{Substitute.}\\
&= -(\log 6.3 + \log 10^{-5}) && \text{Product property}\\
&= -\log 6.3 - (-5) && \text{Distributive property; } \log 10^n = n\\
&= -\log 6.3 + 5 && \text{Definition of subtraction}\\
\text{pH} &\approx 4.2 && \text{Use a calculator.}
\end{aligned}$$

Since the pH is between 3.0 and 6.0, the wetland is a poor fen.

✔ ***Now Try Exercise 37.***

EXAMPLE 4 Measuring the Loudness of Sound

The loudness of sounds is measured in **decibels.** We first assign an intensity of I_0 to a very faint **threshold sound.** If a particular sound has intensity I, then the decibel rating d of this louder sound is given by the following formula.

$$d = 10 \log \frac{I}{I_0}$$

Find the decibel rating d of a sound with intensity $10{,}000I_0$.

SOLUTION

$$\begin{aligned}
d &= 10 \log \frac{10{,}000I_0}{I_0} && \text{Let } I = 10{,}000I_0.\\
&= 10 \log 10{,}000 && \frac{I_0}{I_0} = 1\\
&= 10(4) && \log 10{,}000 = \log 10^4 = 4 \text{ (Section 4.3)}\\
&= 40 && \text{Multiply.}
\end{aligned}$$

The sound has a decibel rating of 40.

✔ ***Now Try Exercise 63.***

LOOKING AHEAD TO CALCULUS

The natural logarithmic function $f(x) = \ln x$ and the reciprocal function $g(x) = \frac{1}{x}$ have an important relationship in calculus. The derivative of the natural logarithmic function is the reciprocal function. Using **Leibniz notation** (named after one of the co-inventors of calculus), we write this fact as $\frac{d}{dx}(\ln x) = \frac{1}{x}$.

Natural Logarithms In **Section 4.2,** we introduced the irrational number e. In most practical applications of logarithms, e is used as base. Logarithms with base e are called **natural logarithms,** since they occur in the life sciences and economics in natural situations that involve growth and decay. The base e logarithm of x is written **ln *x*** (read **"el-en *x*"**). ***The expression* ln *x* *represents the exponent to which e must be raised to obtain x.***

Natural Logarithm

For all positive numbers x,

$$\ln x = \log_e x.$$

A graph of the natural logarithmic function $f(x) = \ln x$ is given in **Figure 35.**

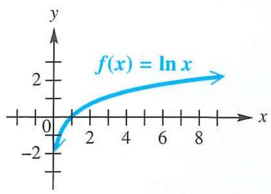


Figure 35

EXAMPLE 5 Evaluating Natural Logarithms with a Calculator

Use a calculator to find the values of

$$\ln e^3, \quad \ln 142, \quad \text{and} \quad \ln 0.005832.$$

SOLUTION **Figure 36** shows that the exact value of $\ln e^3$ is 3, and that

$$\ln 142 \approx 4.955827058$$

and $$\ln 0.005832 \approx -5.144395284.$$

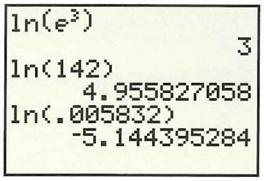


Figure 36

✔ ***Now Try Exercises 45, 49, and 51.***

Figure 37 illustrates that **ln *x*** ***is the exponent to which e must be raised to obtain x.***

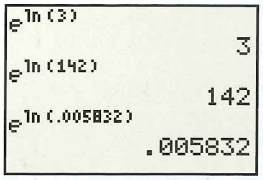


Figure 37

Applications and Models with Natural Logarithms We now consider two applications of natural logarithms.

EXAMPLE 6 Measuring the Age of Rocks

Geologists sometimes measure the age of rocks by using "atomic clocks." By measuring the amounts of potassium-40 and argon-40 in a rock, it is possible to find the age t of the specimen in years with the formula

$$t = (1.26 \times 10^9)\frac{\ln\left(1 + 8.33\left(\frac{A}{K}\right)\right)}{\ln 2},$$

where A and K are the numbers of atoms of argon-40 and potassium-40, respectively, in the specimen.

(a) How old is a rock in which $A = 0$ and $K > 0$?

(b) The ratio $\frac{A}{K}$ for a sample of granite from New Hampshire is 0.212. How old is the sample?

SOLUTION

(a) If $A = 0$, then $\frac{A}{K} = 0$ and the equation is as follows.

$$t = (1.26 \times 10^9)\frac{\ln\left(1 + 8.33\left(\frac{A}{K}\right)\right)}{\ln 2} \quad \text{Given formula}$$

$$= (1.26 \times 10^9)\frac{\ln 1}{\ln 2} \quad \tfrac{A}{K} = 0, \text{ so } \ln(1+0) = \ln 1$$

$$= (1.26 \times 10^9)(0) \quad \ln 1 = 0$$

$$t = 0$$

The rock is new (0 yr old).

(b) Since $\frac{A}{K} = 0.212$, we have the following.

$$t = (1.26 \times 10^9)\frac{\ln(1 + 8.33(0.212))}{\ln 2} \quad \text{Substitute.}$$

$$t \approx 1.85 \times 10^9 \quad \text{Use a calculator.}$$

The granite is about 1.85 billion yr old.

✔ ***Now Try Exercise 77.***

EXAMPLE 7 Modeling Global Temperature Increase

Carbon dioxide in the atmosphere traps heat from the sun. The additional solar radiation trapped by carbon dioxide is called **radiative forcing.** It is measured in watts per square meter (w/m^2). In 1896 the Swedish scientist Svante Arrhenius modeled radiative forcing R caused by additional atmospheric carbon dioxide, using the logarithmic equation

$$R = k \ln \frac{C}{C_0},$$

where C_0 is the preindustrial amount of carbon dioxide, C is the current carbon dioxide level, and k is a constant. Arrhenius determined that $10 \le k \le 16$ when $C = 2C_0$. (*Source:* Clime, W., *The Economics of Global Warming,* Institute for International Economics, Washington, D.C.)

(a) Let $C = 2C_0$. Is the relationship between R and k linear or logarithmic?

(b) The average global temperature increase T (in °F) is given by $T(R) = 1.03R$. Write T as a function of k.

SOLUTION

(a) If $C = 2C_0$, then $\frac{C}{C_0} = 2$, so $R = k \ln 2$ is a linear relation, because $\ln 2$ is a constant.

(b)

$$T(R) = 1.03R$$

$$T(k) = 1.03k \ln \frac{C}{C_0} \quad \text{Use the given expression for } R.$$

✔ ***Now Try Exercise 75.***

Logarithms with Other Bases We can use a calculator to find the values of either natural logarithms (base e) or common logarithms (base 10). However, sometimes we must use logarithms with other bases. The change-of-base theorem can be used to convert logarithms from one base to another.

LOOKING AHEAD TO CALCULUS

In calculus, natural logarithms are more convenient to work with than logarithms with other bases. The change-of-base theorem enables us to convert any logarithmic function to a *natural* logarithmic function.

Change-of-Base Theorem

For any positive real numbers x, a, and b, where $a \neq 1$ and $b \neq 1$, the following holds.

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Proof Let

$$\begin{aligned} y &= \log_a x. & \\ a^y &= x & \text{Change to exponential form.} \\ \log_b a^y &= \log_b x & \text{Take the base } b \text{ logarithm on each side.} \\ y \log_b a &= \log_b x & \text{Power property (Section 4.3)} \\ y &= \frac{\log_b x}{\log_b a} & \text{Divide each side by } \log_b a. \\ \log_a x &= \frac{\log_b x}{\log_b a} & \text{Substitute } \log_a x \text{ for } y. \end{aligned}$$

Any positive number other than 1 can be used for base b in the change-of-base theorem, but usually the only practical bases are e and 10 since calculators give logarithms for these two bases.

For example, with the change-of-base theorem, we can now graph the equation $y = \log_2 x$ by directing the calculator to graph $y = \frac{\log x}{\log 2}$, or, equivalently, $y = \frac{\ln x}{\ln 2}$. ■

EXAMPLE 8 Using the Change-of-Base Theorem

Use the change-of-base theorem to find an approximation to four decimal places for each logarithm.

(a) $\log_5 17$ **(b)** $\log_2 0.1$

SOLUTION

(a) We will arbitrarily use natural logarithms.

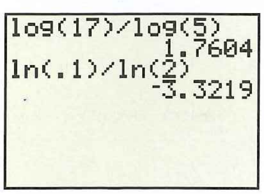
There is no need to actually write this step.

$$\log_5 17 = \frac{\ln 17}{\ln 5} \approx \frac{2.8332}{1.6094} \approx 1.7604$$

(b) Here, we use common logarithms.

$$\log_2 0.1 = \frac{\log 0.1}{\log 2} \approx -3.3219$$

✔ ***Now Try Exercises 79 and 81.***



The screen shows how the result of **Example 8(a)** can be found using *common* logarithms, and how the result of **Example 8(b)** can be found using *natural* logarithms. The results are the same as those in **Example 8.**

NOTE In **Example 8,** logarithms evaluated in the intermediate steps, such as ln 17 and ln 5, were shown to four decimal places. However, the final answers were obtained *without* rounding these intermediate values, using all the digits obtained with the calculator. ***In general, it is best to wait until the final step to round off the answer; otherwise, a build-up of round-off errors may cause the final answer to have an incorrect digit in the final decimal place.***

EXAMPLE 9 Modeling Diversity of Species

One measure of the diversity of the species in an ecological community is modeled by the formula

$$H = -[P_1 \log_2 P_1 + P_2 \log_2 P_2 + \cdots + P_n \log_2 P_n],$$

where $P_1, P_2, \ldots, P_n$ are the proportions of a sample that belong to each of n species found in the sample. (*Source:* Ludwig, J., and J. Reynolds, *Statistical Ecology: A Primer on Methods and Computing,* New York, Wiley.)

Find the measure of diversity in a community with two species where there are 90 of one species and 10 of the other.

SOLUTION Since there are 100 members in the community, $P_1 = \frac{90}{100} = 0.9$ and $P_2 = \frac{10}{100} = 0.1$, so

$$H = -[0.9 \log_2 0.9 + 0.1 \log_2 0.1]. \quad \text{Substitute for } P_1 \text{ and } P_2.$$

In **Example 8(b),** we found that $\log_2 0.1 \approx -3.32$. Now we find $\log_2 0.9$.

$$\log_2 0.9 = \frac{\log 0.9}{\log 2} \approx -0.152 \quad \text{Change-of-base theorem}$$

Therefore,

$$H = -[0.9 \log_2 0.9 + 0.1 \log_2 0.1]$$
$$H \approx -[0.9(-0.152) + 0.1(-3.32)] \quad \text{Substitute approximate values.}$$
$$H \approx 0.469. \quad \text{Simplify.}$$

Verify that $H \approx 0.971$ if there are 60 of one species and 40 of the other. As the proportions of n species get closer to $\frac{1}{n}$ each, the measure of diversity increases to a maximum of $\log_2 n$.

✔ ***Now Try Exercise 73.***

At the end of **Section 4.2,** we saw that graphing calculators are capable of fitting exponential curves to data that suggest such behavior. The same is true for logarithmic curves. For example, during the early 2000s on one particular day, interest rates for various U.S. Treasury Securities were as shown in the table.

Time	3-mo	6-mo	2-yr	5-yr	10-yr	30-yr
Yield	0.83%	0.91%	1.35%	2.46%	3.54%	4.58%

Source: U.S. Treasury.

Figure 38 shows how a calculator gives the best-fitting natural logarithmic curve for the data, as well as the data points and the graph of this curve.

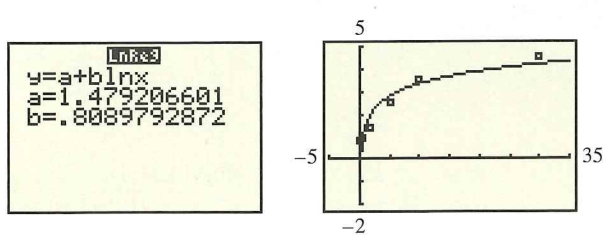


Figure 38

4.4 Exercises

Concept Check Answer each of the following.

1. For the exponential function $f(x) = a^x$, where $a > 1$, is the function increasing or decreasing over its entire domain?

2. For the logarithmic function $g(x) = \log_a x$, where $a > 1$, is the function increasing or decreasing over its entire domain?

3. If $f(x) = 5^x$, what is the rule for $f^{-1}(x)$?

4. What is the name given to the exponent to which 4 must be raised to obtain 11?

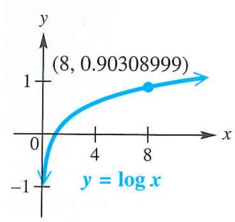
5. A base e logarithm is called a(n) ________ logarithm, and a base 10 logarithm is called a(n) ________ logarithm.

6. How is $\log_3 12$ written in terms of natural logarithms?

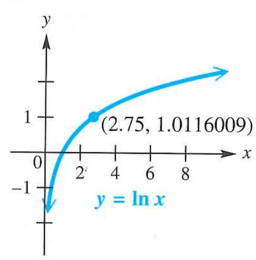
7. Why is $\log_2 0$ undefined?

8. Between what two consecutive integers must $\log_2 12$ lie?

9. The graph of $y = \log x$ shows a point on the graph. Write the logarithmic equation associated with that point.



10. The graph of $y = \ln x$ shows a point on the graph. Write the logarithmic equation associated with that point.



Find each value. If applicable, give an approximation to four decimal places. ***See Example 1.***

11. $\log 10^{12}$
12. $\log 10^7$
13. $\log 0.1$
14. $\log 0.01$
15. $\log 63$
16. $\log 94$
17. $\log 0.0022$
18. $\log 0.0055$
19. $\log(387 \times 23)$
20. $\log(296 \times 12)$
21. $\log\left(\frac{518}{342}\right)$
22. $\log\left(\frac{643}{287}\right)$
23. $\log 387 + \log 23$
24. $\log 296 + \log 12$
25. $\log 518 - \log 342$
26. $\log 643 - \log 287$
27. Explain why the result in **Exercise 23** is the same as that in **Exercise 19.**
28. Explain why the result in **Exercise 25** is the same as that in **Exercise 21.**

For each substance, find the pH from the given hydronium ion concentration. ***See Example 2(a).***

29. grapefruit, 6.3×10^{-4}

30. limes, 1.6×10^{-2}

31. crackers, 3.9×10^{-9}

32. sodium hydroxide (lye), 3.2×10^{-14}

Find the $[H_3O^+]$ *for each substance with the given pH.* ***See Example 2(b).***

33. soda pop, 2.7

34. wine, 3.4

35. beer, 4.8

36. drinking water, 6.5

In Exercises 37–42, suppose that water from a wetland area is sampled and found to have the given hydronium ion concentration. Determine whether the wetland is a rich fen, a poor fen, or a bog. ***See Example 3.***

37. 2.49×10^{-5}

38. 6.22×10^{-5}

39. 2.49×10^{-2}

40. 3.14×10^{-2}

41. 2.49×10^{-7}

42. 5.86×10^{-7}

43. Use your calculator to find an approximation for each logarithm.

(a) log 398.4 **(b)** log 39.84 **(c)** log 3.984

(d) From your answers to parts (a)–(c), make a conjecture concerning the decimal values in the approximations of common logarithms of numbers greater than 1 that have the same digits.

44. Given that $\log 25 \approx 1.3979$, $\log 250 \approx 2.3979$, and $\log 2500 \approx 3.3979$, make a conjecture for an approximation of log 25,000. Then explain why this pattern continues.

Find each value. If applicable, give an approximation to four decimal places. ***See Example 5.***

45. $\ln e^{1.6}$

46. $\ln e^{5.8}$

47. $\ln\left(\frac{1}{e^2}\right)$

48. $\ln\left(\frac{1}{e^4}\right)$

49. ln 28

50. ln 39

51. ln 0.00013

52. ln 0.0077

53. $\ln(27 \times 943)$

54. $\ln(33 \times 568)$

55. $\ln\left(\frac{98}{13}\right)$

56. $\ln\left(\frac{84}{17}\right)$

57. ln 27 + ln 943

58. ln 33 + ln 568

59. ln 98 − ln 13

60. ln 84 − ln 17

61. Explain why the result in **Exercise 57** is the same as that in **Exercise 53.**

62. Explain why the result in **Exercise 59** is the same as that in **Exercise 55.**

Solve each application of logarithms. ***See Examples 4, 6, 7, and 9.***

63. ***Decibel Levels*** Find the decibel ratings of sounds having the following intensities.

(a) $100I_0$ **(b)** $1000I_0$ **(c)** $100{,}000I_0$ **(d)** $1{,}000{,}000I_0$

(e) If the intensity of a sound is doubled, by how much is the decibel rating increased?

64. ***Decibel Levels*** Find the decibel ratings of the following sounds, having intensities as given. Round each answer to the nearest whole number.

(a) whisper, $115I_0$ **(b)** busy street, $9{,}500{,}000I_0$

(c) heavy truck, 20 m away, $1{,}200{,}000{,}000I_0$

(d) rock music, $895{,}000{,}000{,}000I_0$

(e) jetliner at takeoff, $109{,}000{,}000{,}000{,}000I_0$

65. ***Earthquake Intensity*** The magnitude of an earthquake, measured on the Richter scale, is $\log_{10} \frac{I}{I_0}$, where I is the amplitude registered on a seismograph 100 km from the epicenter of the earthquake, and I_0 is the amplitude of an earthquake of a certain (small) size. Find the Richter scale ratings for earthquakes having the following amplitudes.

(a) $1000I_0$ **(b)** $1{,}000{,}000I_0$ **(c)** $100{,}000{,}000I_0$

66. ***Earthquake Intensity*** On December 26, 2004, an earthquake struck in the Indian Ocean with a magnitude of 9.1 on the Richter scale. The resulting tsunami killed an estimated 229,900 people in several countries. Express this reading in terms of I_0.

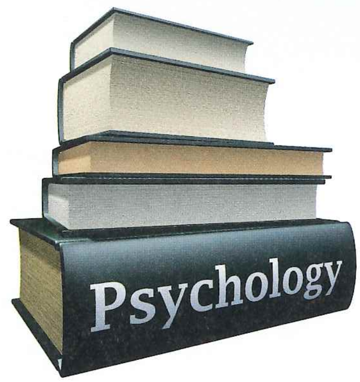
67. ***Earthquake Intensity*** On February 27, 2010, a massive earthquake struck Chile with a magnitude of 8.8 on the Richter scale. Express this reading in terms of I_0.

68. ***Earthquake Intensity Comparison*** Compare your answers to **Exercises 66 and 67.** How many times greater was the force of the 2004 earthquake than that of the 2010 earthquake?

69. ***(Modeling) Bachelor's Degrees in Psychology*** The table gives the number of bachelor's degrees in psychology (in thousands) earned at U.S. colleges and universities for selected years from 1980 through 2008. Suppose x represents the number of years since 1950. Thus, 1980 is represented by 30, 1990 is represented by 40, and so on.

Year	Degrees Earned (in thousands)
1980	42.1
1990	54.0
2000	74.2
2005	85.6
2007	90.0
2008	92.6

Source: U.S. National Center for Education Statistics.



The logarithmic function

$$f(x) = -228.1 + 78.19 \ln x$$

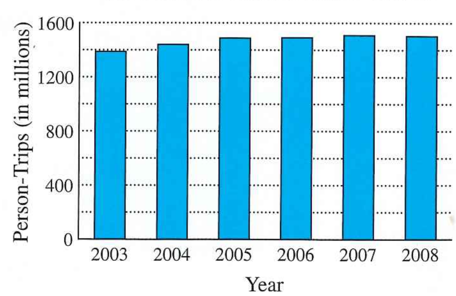
is the best-fitting logarithmic model for the data. Use this function to estimate the number of bachelor's degrees in psychology earned in the year 2012. What assumption must we make to estimate the number of degrees in years beyond 2012?

70. ***(Modeling) Domestic Leisure Travel*** The bar graph shows numbers of leisure trips within the United States (in millions of person-trips of 50 or more miles one-way) over the years 2003–2008. The function

$$f(t) = 1393 + 69.49 \ln t, \quad t \geq 1,$$

where t represents the number of years since 2002 and $f(t)$ is the number of person-trips, in millions, approximates the curve reasonably well.

U.S. Domestic Leisure Travel Volume



Source: U.S. Travel Association.

(a) Use the function to approximate the number of person-trips in 2006. How does this approximation compare to the actual number of 1492 million?

(b) Explain why an exponential function would *not* provide a good model for these data.

71. *(Modeling) Diversity of Species* The number of species $S(n)$ in a sample is given by

$$S(n) = a \ln\left(1 + \frac{n}{a}\right),$$

where n is the number of individuals in the sample, and a is a constant that indicates the diversity of species in the community. If $a = 0.36$, find $S(n)$ for each value of n. (*Hint:* $S(n)$ must be a whole number.)

(a) 100 **(b)** 200 **(c)** 150 **(d)** 10

72. *(Modeling) Diversity of Species* In **Exercise 71,** find $S(n)$ if a changes to 0.88. Use the following values of n.

(a) 50 **(b)** 100 **(c)** 250

73. *(Modeling) Diversity of Species* Suppose a sample of a small community shows two species with 50 individuals each. Find the measure of diversity H.

74. *(Modeling) Diversity of Species* A virgin forest in northwestern Pennsylvania has 4 species of large trees with the following proportions of each: hemlock, 0.521; beech, 0.324; birch, 0.081; maple, 0.074. Find the measure of diversity H.

75. *(Modeling) Global Temperature Increase* In **Example 7,** we expressed the average global temperature increase T (in °F) as

$$T(k) = 1.03k \ln \frac{C}{C_0},$$

where C_0 is the preindustrial amount of carbon dioxide, C is the current carbon dioxide level, and k is a constant. Arrhenius determined that $10 \le k \le 16$ when C was double the value C_0. Use $T(k)$ to find the range of the rise in global temperature T (rounded to the nearest degree) that Arrhenius predicted. (*Source:* Clime, W., *The Economics of Global Warming,* Institute for International Economics, Washington, D.C.)

76. *(Modeling) Global Temperature Increase* (Refer to **Exercise 75.**) According to one study by the IPCC, future increases in average global temperatures (in °F) can be modeled by

$$T(C) = 6.489 \ln \frac{C}{280},$$

where C is the concentration of atmospheric carbon dioxide (in ppm). C can be modeled by the function

$$C(x) = 353(1.006)^{x-1990},$$

where x is the year. (*Source:* International Panel on Climate Change (IPCC).)

(a) Write T as a function of x.

(b) Using a graphing calculator, graph $C(x)$ and $T(x)$ on the interval [1990, 2275] using different coordinate axes. Describe the graph of each function. How are C and T related?

(c) Approximate the slope of the graph of T. What does this slope represent?

(d) Use graphing to estimate x and $C(x)$ when $T(x) = 10$°F.

77. *Age of Rocks* Use the formula of **Example 6** to estimate the age of a rock sample having $\frac{A}{K} = 0.103$.

78. ***(Modeling) Planets' Distances from the Sun and Periods of Revolution*** The table contains the planets' average distances D from the sun and their periods P of revolution around the sun in years. The distances have been normalized so that Earth is one unit away from the sun. For example, since Jupiter's distance is 5.2, its distance from the sun is 5.2 times farther than Earth's.

Planet	D	P
Mercury	0.39	0.24
Venus	0.72	0.62
Earth	1	1
Mars	1.52	1.89
Jupiter	5.2	11.9
Saturn	9.54	29.5
Uranus	19.2	84.0
Neptune	30.1	164.8

Source: Ronan, C., *The Natural History of the Universe,* MacMillan Publishing Co., New York.

(a) Using a graphing calculator, make a scatter diagram by plotting the point $(\ln D, \ln P)$ for each planet on the xy-coordinate axes. Do the data points appear to be linear?

(b) Determine a linear equation that models the data points. Graph your line and the data on the same coordinate axes.

(c) Use this linear model to predict the period of Pluto if its distance is 39.5. Compare your answer to the actual value of 248.5 yr.

Use the change-of-base theorem to find an approximation to four decimal places for each logarithm. ***See Example 8.***

79. $\log_2 5$ **80.** $\log_2 9$ **81.** $\log_8 0.59$ **82.** $\log_8 0.71$

83. $\log_{1/2} 3$ **84.** $\log_{1/3} 2$ **85.** $\log_\pi e$ **86.** $\log_\pi \sqrt{2}$

87. $\log_{\sqrt{13}} 12$ **88.** $\log_{\sqrt{19}} 5$ **89.** $\log_{0.32} 5$ **90.** $\log_{0.91} 8$

Let $u = \ln a$ and $v = \ln b$. Write each expression in terms of u and v without using the ln *function.*

91. $\ln\left(b^4\sqrt{a}\right)$ **92.** $\ln \frac{a^3}{b^2}$ **93.** $\ln\sqrt{\frac{a^3}{b^5}}$ **94.** $\ln\left(\sqrt[3]{a} \cdot b^4\right)$

Concept Check *In Exercises 95–98, use the various properties of exponential and logarithmic functions to evaluate the expressions in parts (a)–(c).*

95. Given $g(x) = e^x$, find **(a)** $g(\ln 4)$ **(b)** $g(\ln(5^2))$ **(c)** $g\left(\ln\left(\frac{1}{e}\right)\right)$.

96. Given $f(x) = 3^x$, find **(a)** $f(\log_3 2)$ **(b)** $f(\log_3(\ln 3))$ **(c)** $f(\log_3(2 \ln 3))$.

97. Given $f(x) = \ln x$, find **(a)** $f(e^6)$ **(b)** $f(e^{\ln 3})$ **(c)** $f(e^{2 \ln 3})$.

98. Given $f(x) = \log_2 x$, find **(a)** $f(2^7)$ **(b)** $f(2^{\log_2 2})$ **(c)** $f(2^{2 \log_2 2})$.

Work each problem.

99. ***Concept Check*** Which of the following is equivalent to $2 \ln(3x)$ for $x > 0$?

A. $\ln 9 + \ln x$ **B.** $\ln(6x)$ **C.** $\ln 6 + \ln x$ **D.** $\ln(9x^2)$

100. ***Concept Check*** Which of the following is equivalent to $\ln(4x) - \ln(2x)$ for $x > 0$?

A. $2 \ln x$ **B.** $\ln(2x)$ **C.** $\frac{\ln(4x)}{\ln(2x)}$ **D.** $\ln 2$

101. The function $f(x) = \ln|x|$ plays a prominent role in calculus. Find its domain, its range, and the symmetries of its graph.

102. Consider the function $f(x) = \log_3 |x|$.

(a) What is the domain of this function?

(b) Use a graphing calculator to graph $f(x) = \log_3 |x|$ in the window $[-4, 4]$ by $[-4, 4]$.

(c) How might one easily misinterpret the domain of the function by merely observing the calculator graph?