Different Methods of Proof

Lesson Plan – Paragraph Proofs

Essential Questions

How can deductive reasoning be used to validate conjectures?

What methods can be used to organize a deductive argument?

Warm-Up/Opening Activity

Construct a flow chart demonstrating the steps taken to get ready for school.

Development of Ideas

Convert the boxes of the flowchart to sentences and form a paragraph showing the steps taken to get ready for school.

Convert flow-chart proofs to paragraph proofs.

Justify geometric properties in paragraph form.

Worksheet: **Introduction to Paragraph Proofs**

Answers:

1. a. The first sentence contains the given statements.
   b. The last sentence contains what is to be proved.
2. Lines $\overline{AB}$ and $\overline{CD}$ are parallel and E is the midpoint of $\overline{AD}$. Since the $\overline{AB}$ and $\overline{CD}$ are parallel, angles BAE and CDE are congruent because if two parallel lines are cut by a transversal, then alternate interior angles are congruent. For this same reason, angles ABE and DCE are congruent. Since E is the midpoint of $\overline{AD}$, $\overline{AE}$ and $\overline{ED}$ are congruent. Therefore, by angle-angle-side triangle congruence, ΔABE is congruent to ΔDCE.
3. Since $\overline{\text{DG}} \parallel \overline{\text{EF}}$ and $\overline{\text{DE}} \parallel \overline{\text{GF}}$ are given, then $\angle 1 \cong \angle 4$ and $\angle 3 \cong \angle 2$ because when two parallel lines are cut by a transversal, then alternate interior angles are congruent. $\overline{\text{DF}} \cong \overline{\text{DF}}$ because of the reflexive property of congruence. Then $\Delta\text{DGF} \cong \Delta\text{FED}$ by angle-side-angle triangle congruence. Therefore, $\overline{\text{DG}} \cong \overline{\text{EF}}$ by the definition of triangle congruence.

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Development of Ideas (Continued)

Answers to Introduction to Paragraph Proofs (Continued)

4. Since $\overline{AC}$ bisects $\angle BAD$ is given, then $\angle BAC \cong \angle CAD$ because of the definition of angle bisectors. Since $\overline{AC}$ bisects $\angle BCD$ is given, then $\angle BCA \cong \angle ACD$ because of the definition of angle bisectors. $\overline{AC} \cong \overline{AC}$ because of the reflexive property of congruence. Then, $\Delta BAC \cong \Delta DAC$ by the angle-side-angle triangle congruence theorem. Therefore $\overline{AB} \cong \overline{AD}$ by the definition of triangle congruence.
5. Since $\angle E$ and $\angle S$ are right angles, they both measure 90 degrees by the definition of right angles. Because of this, they are also congruent. We are also given that $\overline{EF} \cong \overline{ST}$ and $\overline{ED} \cong \overline{SR}$. Because of this information, $\Delta DEF \cong \Delta RST$ because of side-angle-side triangle congruence.

Justify geometric properties in paragraph form.

Worksheet: **Paragraph Proofs**

Answers:

1. $\angle A$ is congruent to $\angle B$ and $\angle A$ is supplementary to $\angle B$. Since the two angles are supplementary, their sum is 180º. Since they are congruent, they can be substituted for one another, meaning that $\angle A + \angle B$ is equal to 180º, but also that 2 times ($\angle B$) is equal to 180º. Then, $\angle B = 90º$ by the division property of equality. Since the two angles are congruent, $\angle A$ also = 90º. $\angle A$ and $\angle B$ are right angles by the definition of right angles.
2. $\angle 1$ and $\angle 3$ are vertical angles. Since they are vertical angles, there is an angle in between them, $\angle 2$, which is adjacent to both angles and supplementary to both angles. Since both $\angle 1$ and $\angle 3$ are supplementary to $\angle 2$, $\angle 1 + \angle 2 = 180º$ and $\angle 2 + \angle 3 = 180º$. $\angle 1 + \angle 2 = \angle 2 + \angle 3$ by the application of the transitive property of equality. $\angle 1$ and $\angle 3$ are congruent because to the subtraction property of equality.
3. ABCD is a rectangle with $\overline{AC}$ and $\overline{BD}$ as diagonals. Since ABCD is a rectangle, opposite sides $\overline{AB}$ and $\overline{CD}$ are congruent. In addition, $\overline{BC}$ and $\overline{DA}$ are congruent. Since

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Development of Ideas (Continued)

Answers to Paragraph Proofs (Continued)

3. (cont.) ABCD is a rectangle, $\angle B$ and $\angle C$ are right angles, and both equal to 90º by the definition of right angles. Since both are equal to 90º, they are equal to one another by the transitive property. ΔABC and ΔBCD are congruent by the side-angle-side triangle congruence theorem. $\overline{AC}$ is congruent to $\overline{BD}$ by the definition of congruent triangles.
4. ABDC is a rhombus with diagonals $\overline{AC}$ and $\overline{BD}$. Since ABCD is a rhombus, all four sides are congruent. In addition, $\overline{AC} \cong \overline{AC}$ and $\overline{BD} \cong \overline{BD}$ by the reflexive property of congruence. ΔABC is congruent to ΔCDA and ΔBCD is congruent to ΔDAB by the side-side-side triangle congruence theorem. Therefore,

   $\angle BAC \cong \angle DAC$ $\quad$ $\angle BCA \cong \angle DCA$

   $\angle ABD \cong \angle CBD$ $\quad$ $\angle CDA \cong \angle ADC$

   by the definition of triangle congruence. $\overline{AC}$ bisects $\angle BAD$ and $\angle BCD$ and $\overline{BD}$ bisects $\angle ADC$ and $\angle ABC$ by the definition of angle bisectors.
5. $\angle B$ is inscribed in circle O and $\overparen{ABC}$ is a semicircle. The measure of arc ABC is 180º by the definition of a semicircle. The $m\angle B$ is 90º because the measure of an inscribed angle is half the measure of its intercepted arc. Therefore, by definition of a right angle, $\angle B$ is a right angle.
6. Quadrilateral ABCD is inscribed in circle O. There are 360º in a circle, so $m\overparen{ABC} + m\overparen{CDA} = 360^\circ$ and the $m\overparen{BCD} + m\overparen{DAB} = 360^\circ$. By the division property of equality, $\frac{1}{2}m\overparen{ABC} + \frac{1}{2}m\overparen{CDA} = 180^\circ$ and

   $$\frac{1}{2}m\overparen{BCD} + \frac{1}{2}m\overparen{DAB} = 180^\circ.$$

   The $m\angle D = \frac{1}{2}m\overparen{ABC}$, $m\angle B = \frac{1}{2}m\overparen{CDA}$,

   $m\angle A = \frac{1}{2}m\overparen{BCD}$, and $m\angle C = \frac{1}{2}m\overparen{DAB}$ because the

Development of Ideas (Continued)

Answers to Paragraph Proofs (Continued)

6. (cont.) measure of an inscribed angle is one-half the measure of its intercepted arc. $m\angle D + m\angle B = 180°$ and $m\angle A + m\angle C = 180°$ by substitution. Therefore, $\angle A$ is supplementary to $\angle C$ and $\angle B$ is supplementary to $\angle D$ by definition of supplementary angles.

7. $\overline{AB}$ is parallel to $\overline{CD}$. Draw $\overline{AD}$. $\angle BAD \cong \angle CDA$ because if two parallel lines are cut by a transversal, the alternate interior angles are congruent. $\angle BAD = \angle CDA$ by the definition of congruent angles. $m\angle BAD = \frac{1}{2} m\overset{\frown}{BD}$ and $m\angle CDA = \frac{1}{2} m\overset{\frown}{AC}$ because the measure of an inscribed angle is one-half the measure of its intercepted arc. $\frac{1}{2} m\overset{\frown}{BD} = \frac{1}{2} m\overset{\frown}{AC}$ by substitution. The measure of $\overset{\frown}{BD} = m\overset{\frown}{AC}$ by the multiplication property of equality. Therefore, $\overset{\frown}{AC} \cong \overset{\frown}{BD}$ by the definition of congruent arcs.

Closure

Explain how deductive reasoning is used in paragraph proofs.

Answer: Deductive reasoning is used to connect the given statements by use of definitions, theorems, and postulates to what is to be proved.

Different Methods of Proof

Introduction to Paragraph Proofs

A **paragraph proof** is another way a proof is often written. The advantage of a paragraph proof is that you have the chance to explain your reasoning in your own words. In a paragraph proof, the statements and their justifications are written together in a logical order in a paragraph form. There is always a diagram and a statement of the given and prove sections before the paragraph.

1. a. What information does the first sentence of a paragraph proof contain?

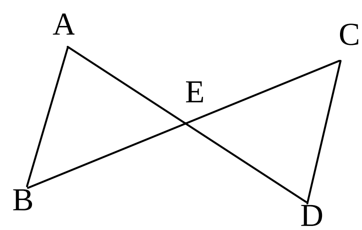
   b. What information does the last sentence of a paragraph proof contain?

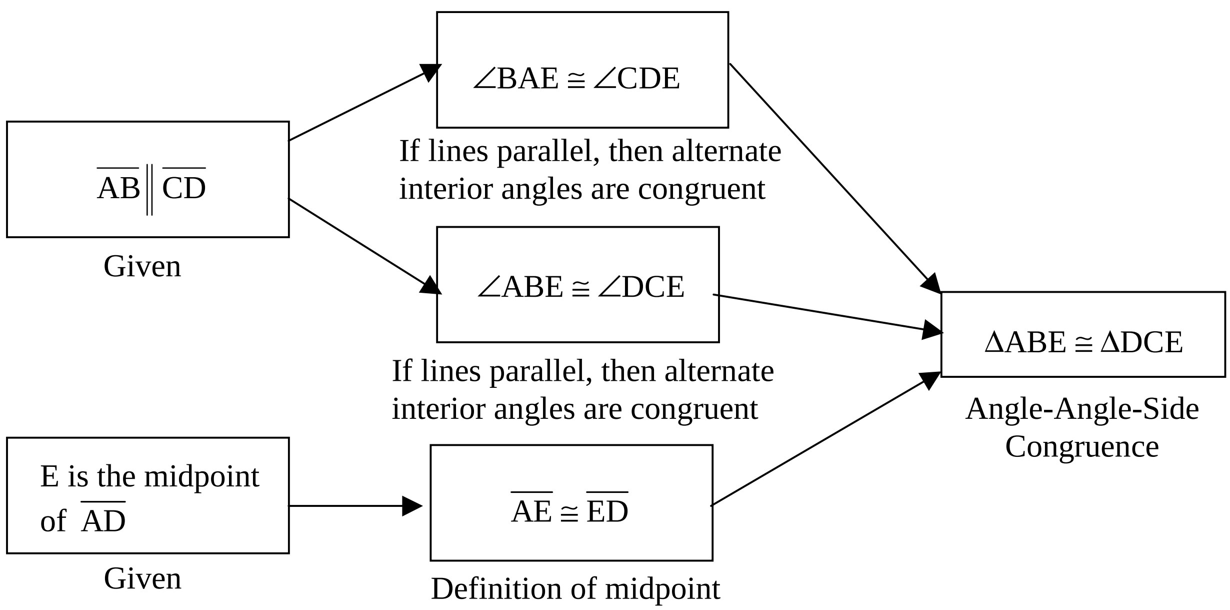
2. For the flow chart proof below, rewrite each box as a statement with the reason for the box as the justification.

Given: $\overline{AB} \parallel \overline{CD}$

E is the midpoint of $\overline{AD}$

Prove: $\Delta ABE \cong \Delta DCE$





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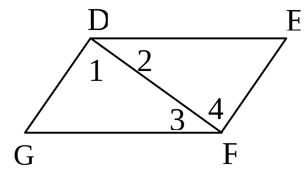
Introduction to Paragraph Proofs (Continued)

3. Fill-in the missing statements and justifications in the following paragraph proof.

Given: $\overline{DG} \parallel \overline{EF}$, $\overline{DE} \parallel \overline{GF}$

Prove: $\overline{DG} \cong \overline{EF}$

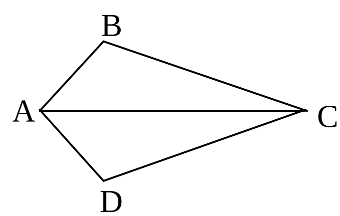


Since $\overline{DG} \parallel \overline{EF}$ and $\overline{DE} \parallel \overline{GF}$ are given, then $\angle 1 \cong \angle 4$ and $\angle 3 \cong \angle$______ because ____. $\overline{DF} \cong \overline{DF}$ because __________________. Then $\Delta DGF \cong \Delta$_____ by ________. Therefore, $\overline{DG} \cong \overline{EF}$ by ___________________________.

4. Fill-in the missing statements and justifications in the following paragraph proof.

Given: $\overline{AC}$ bisects $\angle BAD$
$\overline{AC}$ bisects $\angle BCD$

Prove: $\overline{AB} \cong \overline{AD}$

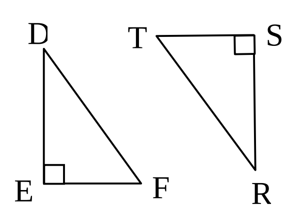


Since $\overline{AC}$ bisects $\angle BAD$ is given, then $\angle BAC \cong \angle$_________ because ________________. Since $\overline{AC}$ bisects $\angle BCD$ is given, then $\angle BCA \cong \angle$______ because ____________. $\overline{AC} \cong \overline{AC}$ because ____________. Then $\Delta BAC \cong \Delta$________ by _______. Therefore $\overline{AB} \cong \overline{AD}$ by _________.

5. Mark the given on the figure. Write your own paragraph proof for the following information.

Given: $\angle E$ and $\angle S$ are right angles.
$\overline{EF} \cong \overline{ST}$ and $\overline{ED} \cong \overline{SR}$

Prove: $\Delta DEF \cong \Delta RST$



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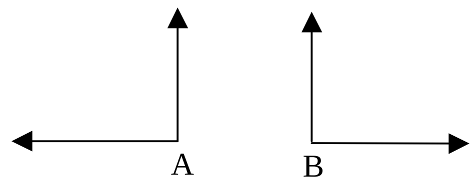
Paragraph Proofs

Use a paragraph proof to justify the following conjectures.

1. If two angles are both congruent and supplementary, then each angle is a right angle.

   Given: $\angle A \cong \angle B$
   $\angle A$ is supplementary to $\angle B$

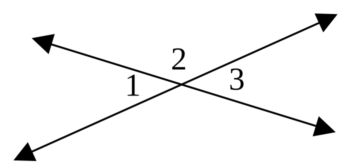
   Prove: $\angle A$ is a right angle
   $\angle B$ is a right angle



2. Vertical angles are congruent.

   Given: $\angle 1$ and $\angle 3$ are vertical angles

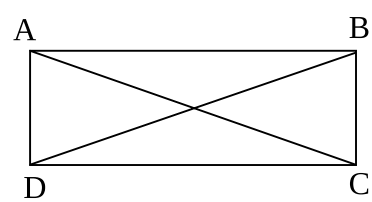
   Prove: $\angle 1 \cong \angle 3$



3. The diagonals of a rectangle are congruent.

   Given: Rectangle ABCD with diagonals $\overline{AC}$ and $\overline{BD}$

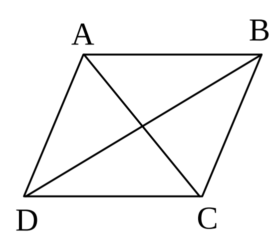
   Prove: $\overline{AC} \cong \overline{BD}$



4. The diagonals of a rhombus bisect the angles.

   Given: Rhombus ABCD with diagonals $\overline{AC}$ and $\overline{BD}$

   Prove: $\overline{AC}$ bisects $\angle BAD$ and $\angle BCD$
   $\overline{BD}$ bisects $\angle ADC$ and $\angle ABC$



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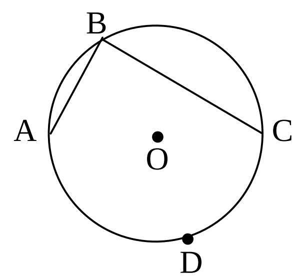
Paragraph Proofs (Continued)

5. Angles inscribed in a semicircle are right angles.

Given: $\angle B$ is inscribed in circle O

$\overset{\frown}{ABC}$ is a semicircle

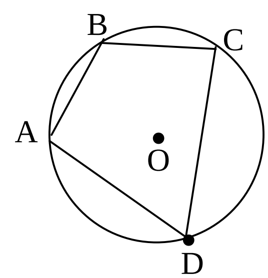
Prove: $\angle B$ is a right angle



6. If a quadrilateral is inscribed in a circle, then the opposite angles are supplementary.

Given: Quadrilateral ABCD is inscribed in circle O

Prove: $\angle A$ is supplementary to $\angle C$
$\angle B$ is supplementary to $\angle D$



7. Parallel lines intercept congruent arcs on a circle.

Given: $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

Prove: $\overset{\frown}{AC} \cong \overset{\frown}{BD}$

(Hint: Draw segment AD)