2.1 Rectangular Coordinates and Graphs

- Ordered Pairs
- The Rectangular Coordinate System
- The Distance Formula
- The Midpoint Formula
- Graphing Equations in Two Variables

Ordered Pairs

The idea of pairing one quantity with another is often encountered in everyday life.

- A numerical score in a mathematics course is paired with a corresponding letter grade.
- The number of gallons of gasoline pumped into a tank is paired with the amount of money needed to purchase it.
- Expense categories are paired with dollars spent by the average American household in 2008. (See the table in the margin.)

Category	Amount Spent
food	\$ 6443
housing	\$17,109
transportation	\$ 8604
health care	\$ 2976
apparel and services	\$ 1801
entertainment	\$ 2835

Source: U.S. Bureau of Labor Statistics.

Pairs of related quantities, such as a 96 determining a grade of A, 3 gallons of gasoline costing \$10.50, and 2008 spending on food of \$6443, can be expressed as *ordered pairs:* (96, A), (3, \$10.50), (food, \$6443). An **ordered pair** consists of two components, written inside parentheses.

EXAMPLE 1 Writing Ordered Pairs

Use the table to write ordered pairs to express the relationship between each category and the amount spent on it.

(a) housing **(b)** entertainment

SOLUTION

(a) Use the data in the second row: (housing, \$17,109).

(b) Use the data in the last row: (entertainment, \$2835).

✔ *Now Try Exercise 9.*

In mathematics, we are most often interested in ordered pairs whose components are numbers. The ordered pairs (a, b) and (c, d) are equal provided that $a = c$ *and* $b = d$.

NOTE Notation such as $(2, 4)$ was used in **Chapter 1** to show an interval on the number line, and the same notation is used to indicate an ordered pair of numbers. The intended use is usually clear from the context of the discussion.

The Rectangular Coordinate System

As mentioned in **Section R.2,** each real number corresponds to a point on a number line. This idea is extended to ordered pairs of real numbers by using two perpendicular number lines, one horizontal and one vertical, that intersect at their zero-points. This point of intersection is called the **origin.** The horizontal line is called the ***x*-axis,** and the vertical line is called the ***y*-axis.**

The x-axis and y-axis together make up a **rectangular coordinate system,** or **Cartesian coordinate system** (named for one of its coinventors, René Descartes. The other coinventor was Pierre de Fermat). The plane into which the coordinate system is introduced is the **coordinate plane,** or ***xy*-plane.** See **Figure 1.** The x-axis and y-axis divide the plane into four regions, or **quadrants,** labeled as shown. The points on the x-axis and y-axis belong to no quadrant.

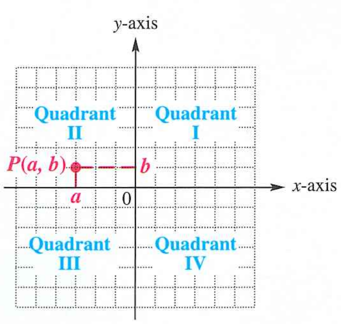


Figure 1

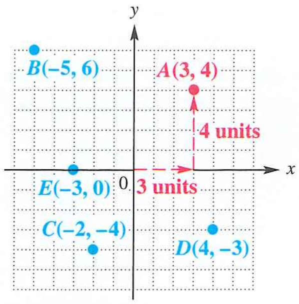


Figure 2

Each point P in the xy-plane corresponds to a unique ordered pair (a, b) of real numbers. The point P corresponding to the ordered pair (a, b) often is written $P(a, b)$ as in **Figure 1** and referred to as "the point (a, b)." The numbers a and b are the **coordinates** of point P. To locate on the xy-plane the point corresponding to the ordered pair $(3, 4)$, for example, start at the origin, move 3 units in the positive x-direction, and then move 4 units in the positive y-direction. See **Figure 2.** Point A corresponds to the ordered pair $(3, 4)$.

The Distance Formula Recall that the distance on a number line between points P and Q with coordinates x_1 and x_2 is

$$d(P, Q) = |x_1 - x_2| = |x_2 - x_1|. \quad \text{Definition of distance (Section R.2)}$$

By using the coordinates of their ordered pairs, we can extend this idea to find the distance between any two points in a plane.

Figure 3 shows the points $P(-4, 3)$ and $R(8, -2)$. We complete a right triangle as in the figure. This right triangle has its 90° angle at $Q(8, 3)$. The legs have lengths

$$d(P, Q) = |8 - (-4)| = 12$$

and $$d(Q, R) = |3 - (-2)| = 5.$$

By the Pythagorean theorem, the hypotenuse has length

$$\sqrt{12^2 + 5^2} = \sqrt{144 + 25} = \sqrt{169} = 13. \quad \text{(Section 1.5)}$$

Thus, the distance between $(-4, 3)$ and $(8, -2)$ is 13.

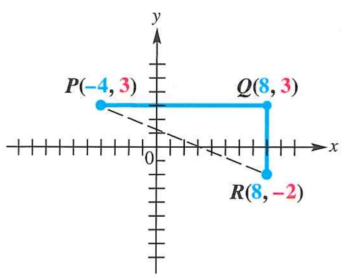


Figure 3

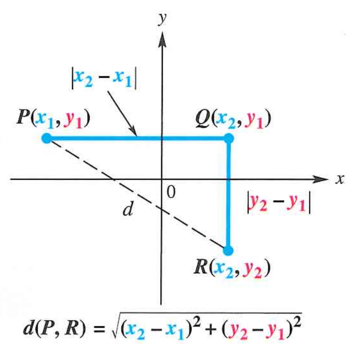


Figure 4

René Descartes (1596–1650)

The initial flash of *analytic geometry* may have come to Descartes as he was watching a fly crawling about on the ceiling near a corner of his room. It struck him that the path of the fly on the ceiling could be described if only one knew the relation connecting the fly's distances from two adjacent walls.

Source: An Introduction to the History of Mathematics by Howard Eves.

To obtain a general formula, let $P(x_1, y_1)$ and $R(x_2, y_2)$ be any two distinct points in a plane, as shown in **Figure 4.** Complete a triangle by locating point Q with coordinates (x_2, y_1). The Pythagorean theorem gives the distance between P and R.

$$d(P, R) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Absolute value bars are not necessary in this formula, since for all real numbers a and b,

$$|a - b|^2 = (a - b)^2.$$

The **distance formula** can be summarized as follows.

Distance Formula

Suppose that $P(x_1, y_1)$ and $R(x_2, y_2)$ are two points in a coordinate plane. The distance between P and R, written $d(P, R)$, is given by the following formula.

$$d(P, R) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

LOOKING AHEAD TO CALCULUS

In analytic geometry and calculus, the distance formula is extended to two points in space. Points in space can be represented by **ordered triples.** The distance between the two points (x_1, y_1, z_1) and (x_2, y_2, z_2) is given by the following expression.

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$$

The distance formula can be stated in words.

The distance between two points in a coordinate plane is the square root of the sum of the square of the difference between their x-coordinates and the square of the difference between their y-coordinates.

Although our derivation of the distance formula assumed that P and R are not on a horizontal or vertical line, the result is true for any two points.

EXAMPLE 2 Using the Distance Formula

Find the distance between $P(-8, 4)$ and $Q(3, -2)$.

SOLUTION Use the distance formula.

$$\begin{aligned} d(P, Q) &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} && \text{Distance formula} \\ &= \sqrt{[3 - (-8)]^2 + (-2 - 4)^2} && x_1 = -8,\ y_1 = 4,\ x_2 = 3,\ y_2 = -2 \\ &= \sqrt{11^2 + (-6)^2} \\ &= \sqrt{121 + 36} \\ &= \sqrt{157} \end{aligned}$$

Be careful when subtracting a negative number.

✔ *Now Try Exercise 11(a).*

A statement of the form "If p, then q" is called a **conditional statement.** The related statement "If q, then p" is called its **converse.** In **Section 1.5** we studied the Pythagorean theorem. Its *converse* is also a true statement.

If the sides a, b, and c of a triangle satisfy $a^2 + b^2 = c^2$, then the triangle is a right triangle with legs having lengths a and b and hypotenuse having length c.

We can use this fact to determine whether three points are the vertices of a right triangle.

EXAMPLE 3 Applying the Distance Formula

Are points $M(-2, 5)$, $N(12, 3)$, and $Q(10, -11)$ the vertices of a right triangle?

SOLUTION A triangle with the three given points as vertices, shown in **Figure 5,** is a right triangle if the square of the length of the longest side equals the sum of the squares of the lengths of the other two sides. Use the distance formula to find the length of each side of the triangle.

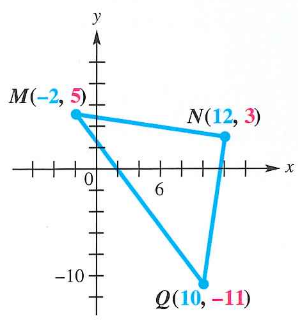


Figure 5

$$d(M, N) = \sqrt{[12 - (-2)]^2 + (3 - 5)^2} = \sqrt{196 + 4} = \sqrt{200}$$

$$d(M, Q) = \sqrt{[10 - (-2)]^2 + (-11 - 5)^2} = \sqrt{144 + 256} = \sqrt{400} = 20$$

$$d(N, Q) = \sqrt{(10 - 12)^2 + (-11 - 3)^2} = \sqrt{4 + 196} = \sqrt{200}$$

The longest side, of length 20 units, is chosen as the possible hypotenuse. Since

$$(\sqrt{200})^2 + (\sqrt{200})^2 = 400 = 20^2$$

is true, the triangle is a right triangle with hypotenuse joining M and Q.

✔ *Now Try Exercise 19.*

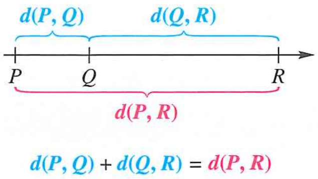


Figure 6

Using a similar procedure, we can tell whether three points are **collinear** (that is, lying on a straight line). ***Three points are collinear if the sum of the distances between two pairs of the points is equal to the distance between the remaining pair of points.*** See **Figure 6.**

EXAMPLE 4 Applying the Distance Formula

Are the points $P(-1, 5)$, $Q(2, -4)$, and $R(4, -10)$ collinear?

SOLUTION

$$d(P, Q) = \sqrt{(-1-2)^2 + [5-(-4)]^2} = \sqrt{9+81} = \sqrt{90} = 3\sqrt{10}$$

(Section R.7)

$$d(Q, R) = \sqrt{(2-4)^2 + [-4-(-10)]^2} = \sqrt{4+36} = \sqrt{40} = 2\sqrt{10}$$

$$d(P, R) = \sqrt{(-1-4)^2 + [5-(-10)]^2} = \sqrt{25+225} = \sqrt{250} = 5\sqrt{10}$$

Because $3\sqrt{10} + 2\sqrt{10} = 5\sqrt{10}$ is true, the three points are collinear.

✔ ***Now Try Exercise 25.***

NOTE In **Exercises 76–80** of **Section 2.5,** we examine another method of determining whether three points are collinear.

The Midpoint Formula The midpoint of a line segment is equidistant from the endpoints of the segment. The **midpoint formula** is used to find the coordinates of the midpoint of a line segment. To develop the midpoint formula, let $P(x_1, y_1)$ and $Q(x_2, y_2)$ be any two distinct points in a plane. (Although **Figure 7** shows $x_1 < x_2$, no particular order is required.) Let $M(x, y)$ be the midpoint of the segment joining P and Q. Draw vertical lines from each of the three points to the x-axis, as shown in **Figure 7.**

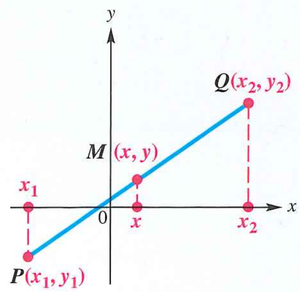


Figure 7

Since $M(x, y)$ is the midpoint of the line segment joining P and Q, the distance between x and x_1 equals the distance between x and x_2.

$$x_2 - x = x - x_1$$

$$x_2 + x_1 = 2x \qquad \text{Add } x \text{ and } x_1. \text{ (Section 1.1)}$$

$$x = \frac{x_1 + x_2}{2} \qquad \text{Divide by 2 and rewrite.}$$

Similarly, the y-coordinate is $\frac{y_1 + y_2}{2}$, yielding the following formula.

Midpoint Formula

The coordinates of the midpoint M of the line segment with endpoints $P(x_1, y_1)$ and $Q(x_2, y_2)$ are given by the following.

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

That is, the x-coordinate of the midpoint of a line segment is the* average *of the x-coordinates of the segment's endpoints, and the y-coordinate is the* average *of the y-coordinates of the segment's endpoints.

EXAMPLE 5 Using the Midpoint Formula

Use the midpoint formula to do each of the following.

(a) Find the coordinates of the midpoint M of the segment with endpoints $(8, -4)$ and $(-6, 1)$.

(b) Find the coordinates of the other endpoint Q of a segment with one endpoint $P(-6, 12)$ and midpoint $M(8, -2)$.

SOLUTION

(a) The coordinates of M are found using the midpoint formula.

$$M = \left(\frac{8 + (-6)}{2}, \frac{-4 + 1}{2}\right) = \left(1, -\frac{3}{2}\right) \quad \text{Substitute in } \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right).$$

The coordinates of midpoint M are $\left(1, -\frac{3}{2}\right)$.

(b) Let (x, y) represent the coordinates of Q. Use the midpoint formula twice.

x-value of P ↓ x-value of M ↓	y-value of P ↓ y-value of M ↓
$\dfrac{x + (-6)}{2} = 8$	$\dfrac{y + 12}{2} = -2$
$x - 6 = 16$	$y + 12 = -4$
$x = 22$	$y = -16$

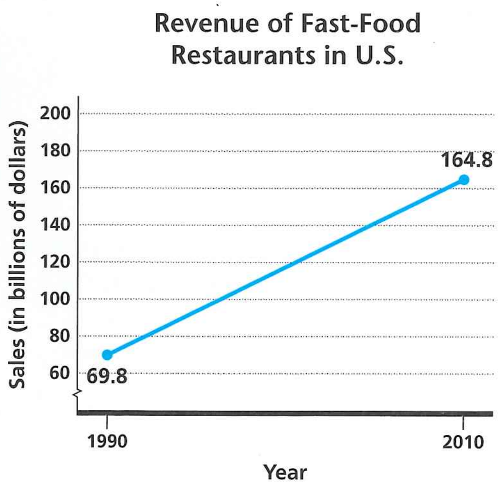
Substitute carefully.

The coordinates of endpoint Q are $(22, -16)$.

✔ *Now Try Exercises 11(b) and 31.*

EXAMPLE 6 Applying the Midpoint Formula

Figure 8 depicts how a graph might indicate the increase in the revenue generated by fast-food restaurants in the United States from \$69.8 billion in 1990 to \$164.8 billion in 2010. Use the midpoint formula and the two given points to estimate the revenue from fast-food restaurants in 2000, and compare it to the actual figure of \$107.1 billion.



Source: National Restaurant Association.

Figure 8

SOLUTION The year 2000 lies halfway between 1990 and 2010, so we must find the coordinates of the midpoint M of the segment that has endpoints

$$(1990, 69.8) \quad \text{and} \quad (2010, 164.8).$$

(Here, the second component is in billions of dollars.)

$$M = \left(\frac{1990 + 2010}{2}, \frac{69.8 + 164.8}{2}\right) = (2000, 117.3) \quad \text{Use the midpoint formula.}$$

Thus, our estimate is \$117.3 billion, which is greater than the actual figure of \$107.1 billion. (This discrepancy is due to actual revenue with an average increase of \$3.73 billion between 1990 and 2000, and then of \$5.77 billion between 2000 and 2010. Graphs such as this can sometimes be misleading.)

✔ *Now Try Exercise 37.*

Graphing Equations in Two Variables Ordered pairs are used to express the solutions of equations in two variables. When an ordered pair represents the solution of an equation with the variables x and y, the x-value is written first. For example, we say that

$$(1, 2) \quad \text{is a solution of} \quad 2x - y = 0,$$

since substituting 1 for x and 2 for y in the equation gives a true statement.

$$2x - y = 0$$

$$2(1) - 2 \stackrel{?}{=} 0 \quad \text{Let } x = 1 \text{ and } y = 2.$$

$$0 = 0 \;\checkmark \quad \text{True}$$

EXAMPLE 7 Finding Ordered-Pair Solutions of Equations

For each equation, find at least three ordered pairs that are solutions.

(a) $y = 4x - 1$ **(b)** $x = \sqrt{y - 1}$ **(c)** $y = x^2 - 4$

SOLUTION

(a) Choose any real number for x or y and substitute in the equation to get the corresponding value of the other variable. For example, let $x = -2$ and then let $y = 3$.

$$y = 4x - 1$$

$$y = 4(-2) - 1 \quad \text{Let } x = -2.$$

$$y = -8 - 1 \quad \text{Multiply.}$$

$$y = -9 \quad \text{Subtract.}$$

$$y = 4x - 1$$

$$3 = 4x - 1 \quad \text{Let } y = 3.$$

$$4 = 4x \quad \text{Add 1.}$$

$$1 = x \quad \text{Divide by 4.}$$

This gives the ordered pairs $(-2, -9)$ and $(1, 3)$. Verify that the ordered pair $(0, -1)$ is also a solution.

(b)

$$x = \sqrt{y - 1} \quad \text{Given equation}$$

$$1 = \sqrt{y - 1} \quad \text{Let } x = 1.$$

$$1 = y - 1 \quad \text{Square each side. (Section 1.6)}$$

$$2 = y \quad \text{Add 1.}$$

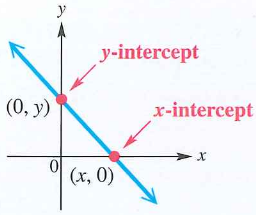
One ordered pair is $(1, 2)$. Verify that the ordered pairs $(0, 1)$ and $(2, 5)$ are also solutions of the equation.

(c) A table provides an organized method for determining ordered pairs. Here, we let x equal -2, -1, 0, 1, and 2 in $y = x^2 - 4$ and determine the corresponding y-values.

x	y	
-2	0	$(-2)^2 - 4 = 4 - 4 = 0$
-1	-3	$(-1)^2 - 4 = 1 - 4 = -3$
0	-4	$0^2 - 4 = -4$
1	-3	$1^2 - 4 = -3$
2	0	$2^2 - 4 = 0$

Five ordered pairs are $(-2, 0)$, $(-1, -3)$, $(0, -4)$, $(1, -3)$, and $(2, 0)$.

✔ *Now Try Exercises 43(a), 47(a), and 49(a).*



The **graph** of an equation is found by plotting ordered pairs that are solutions of the equation. The **intercepts** of the graph are good points to plot first. An **x-intercept** is a point where the graph intersects the x-axis. A **y-intercept** is a point where the graph intersects the y-axis. In other words, the x-intercept is represented by an ordered pair where $y = 0$, and the y-intercept is represented by an ordered pair where $x = 0$.

A general algebraic approach for graphing an equation using intercepts and point-plotting follows.

Graphing an Equation by Point Plotting

Step 1 Find the intercepts.

Step 2 Find as many additional ordered pairs as needed.

Step 3 Plot the ordered pairs from Steps 1 and 2.

Step 4 Join the points from Step 3 with a smooth line or curve.

EXAMPLE 8 Graphing Equations

Graph each of the equations here, from **Example 7.**

(a) $y = 4x - 1$ **(b)** $x = \sqrt{y - 1}$ **(c)** $y = x^2 - 4$

SOLUTION

(a) ***Step 1*** Let $y = 0$ to find the x-intercept, and let $x = 0$ to find the y-intercept.

$y = 4x - 1$	$y = 4x - 1$
$0 = 4x - 1$ Let $y = 0$.	$y = 4(0) - 1$ Let $x = 0$.
$1 = 4x$	$y = 0 - 1$
$\frac{1}{4} = x$	$y = -1$

The intercepts are $\left(\frac{1}{4}, 0\right)$ and $(0, -1)$.* Note that the y-intercept is one of the ordered pairs we found in **Example 7(a).**

*The intercepts are sometimes defined as numbers such as x-intercept $\frac{1}{4}$ and y-intercept -1. In this text, we define them as ordered pairs such as $\left(\frac{1}{4}, 0\right)$ and $(0, -1)$.

Step 2 We use the other ordered pairs found in **Example 7(a):**

$$(-2, -9) \quad \text{and} \quad (1, 3).$$

Step 3 Plot the four ordered pairs from Steps 1 and 2 as shown in **Figure 9.**

Step 4 Join the points plotted in Step 3 with a straight line. This line, also shown in **Figure 9,** is the graph of the equation $y = 4x - 1$.

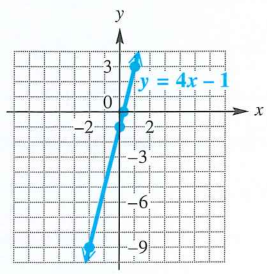


Figure 9

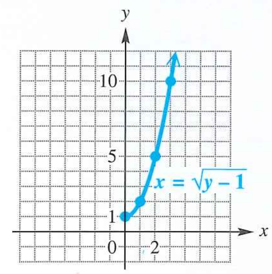


Figure 10

(b) For $x = \sqrt{y - 1}$, the y-intercept $(0, 1)$ was found in **Example 7(b).** Solve

$$x = \sqrt{0 - 1} \qquad \text{Let } y = 0.$$

for the x-intercept. Since the quantity under the radical is negative, there is no x-intercept. In fact, $y - 1$ must be greater than or equal to 0, so y must be greater than or equal to 1.

We start by plotting the ordered pairs from **Example 7(b)** and then join the points with a smooth curve as in **Figure 10.** To confirm the direction the curve will take as x increases, we find another solution, $(3, 10)$. (Point plotting for graphs other than lines is often inefficient. We will examine other graphing methods later.)

(c) In **Example 7(c),** we made a table of five ordered pairs that satisfy the equation $y = x^2 - 4$.

$$\underbrace{(-2, 0)}_{x\text{-intercept}}, \quad (-1, -3), \quad \underbrace{(0, -4)}_{y\text{-intercept}}, \quad (1, -3), \quad \underbrace{(2, 0)}_{x\text{-intercept}}$$

Plotting the points and joining them with a smooth curve gives the graph in **Figure 11.** This curve is called a **parabola.**

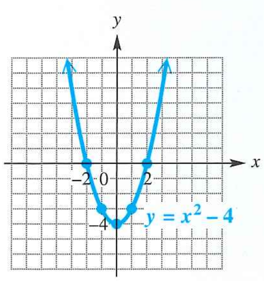


Figure 11

Now Try Exercises 43(b), 47(b), and 49(b).

To graph an equation on a calculator, such as

$$y = 4x - 1, \qquad \text{Equation from } \textbf{Example 8(a)}$$

we must first solve it for y (if necessary). Here the equation is already in the correct form, $y = 4x - 1$, so we enter $4x - 1$ for Y_1.

The intercepts can help determine an appropriate window, since we want them to appear in the graph. A good choice is often the **standard viewing window** for the TI-83/84 Plus, which has X minimum $= -10$, X maximum $= 10$, Y minimum $= -10$, Y maximum $= 10$, with X scale $= 1$ and Y scale $= 1$. (The X and Y scales determine the spacing of the tick marks.) Since the intercepts here are very close to the origin, we have chosen the X and Y minimum and maximum to be -3 and 3 instead. See **Figure 12.** ■

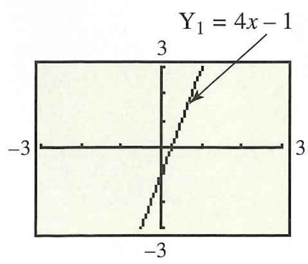


Figure 12

2.1 Exercises

Concept Check *Decide whether each statement in Exercises 1–5 is* true *or* false. *If the statement is false, tell why.*

1. The point $(-1, 3)$ lies in quadrant III of the rectangular coordinate system.
2. The distance from $P(x_1, y_1)$ to $Q(x_2, y_2)$ is given by
$$d(P, Q) = \sqrt{(x_1 - y_1)^2 + (x_2 - y_2)^2}.$$
3. The distance from the origin to the point (a, b) is $\sqrt{a^2 + b^2}$.
4. The midpoint of the segment joining (a, b) and $(3a, -3b)$ has coordinates $(2a, -b)$.
5. The graph of $y = 2x + 4$ has x-intercept $(-2, 0)$ and y-intercept $(0, 4)$.
6. In your own words, list the steps for graphing an equation.

In Exercises 7–10, give three ordered pairs from each table. **See Example 1.**

7.

x	y
2	−5
−1	7
3	−9
5	−17
6	−21

8.

x	y
3	3
−5	−21
8	18
4	6
0	−6

9. ***Percent of High School Students Who Smoke***

Year	Percent
1997	36
1999	35
2001	29
2003	22
2005	23
2007	20

Source: Centers for Disease Control and Prevention.

10. ***Number of U.S. Viewers of the Super Bowl***

Year	Viewers (millions)
1998	90.0
2000	88.5
2002	86.8
2004	89.8
2006	90.7
2008	97.4
2010	106.5

Source: www.tvbythenumbers.com

For the points P and Q, find **(a)** *the distance $d(P, Q)$ and* **(b)** *the coordinates of the midpoint M of the segment PQ.* **See Examples 2 and 5(a).**

11. $P(-5, -7)$, $Q(-13, 1)$
12. $P(-4, 3)$, $Q(2, -5)$
13. $P(8, 2)$, $Q(3, 5)$
14. $P(-8, 4)$, $Q(3, -5)$
15. $P(-6, -5)$, $Q(6, 10)$
16. $P(6, -2)$, $Q(4, 6)$
17. $P(3\sqrt{2}, 4\sqrt{5})$, $Q(\sqrt{2}, -\sqrt{5})$
18. $P(-\sqrt{7}, 8\sqrt{3})$, $Q(5\sqrt{7}, -\sqrt{3})$

Determine whether the three points are the vertices of a right triangle. **See Example 3.**

19. $(-6, -4)$, $(0, -2)$, $(-10, 8)$
20. $(-2, -8)$, $(0, -4)$, $(-4, -7)$
21. $(-4, 1)$, $(1, 4)$, $(-6, -1)$
22. $(-2, -5)$, $(1, 7)$, $(3, 15)$
23. $(-4, 3)$, $(2, 5)$, $(-1, -6)$
24. $(-7, 4)$, $(6, -2)$, $(0, -15)$

Determine whether the three points are collinear. ***See Example 4.***

25. $(0, -7), (-3, 5), (2, -15)$

26. $(-1, 4), (-2, -1), (1, 14)$

27. $(0, 9), (-3, -7), (2, 19)$

28. $(-1, -3), (-5, 12), (1, -11)$

29. $(-7, 4), (6, -2), (-1, 1)$

30. $(-4, 3), (2, 5), (-1, 4)$

Find the coordinates of the other endpoint of each segment, given its midpoint and one endpoint. ***See Example 5(b).***

31. midpoint $(5, 8)$, endpoint $(13, 10)$

32. midpoint $(-7, 6)$, endpoint $(-9, 9)$

33. midpoint $(12, 6)$, endpoint $(19, 16)$

34. midpoint $(-9, 8)$, endpoint $(-16, 9)$

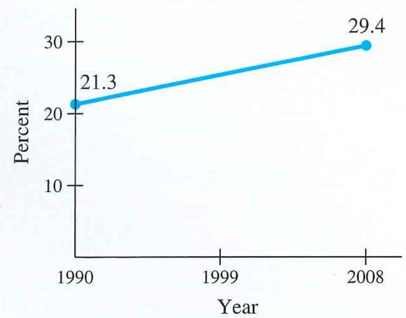
35. midpoint (a, b), endpoint (p, q)

36. midpoint $\left(\frac{a+b}{2}, \frac{c+d}{2}\right)$, endpoint (b, d)

Solve each problem. ***See Example 6.***

37. ***Bachelor's Degree Attainment*** The graph shows a straight line that approximates the percentage of Americans 25 years and older who had earned bachelor's degrees or higher for the years 1990–2008. Use the midpoint formula and the two given points to estimate the percent in 1999. Compare your answer with the actual percent of 25.2.

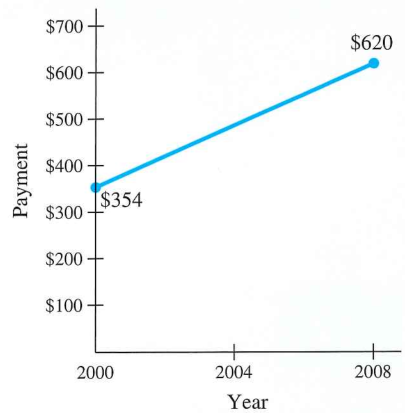
Percent of Bachelor's Degrees or Higher



Source: U.S. Census Bureau.

38. ***Temporary Assistance for Needy Families (TANF)*** The graph shows an idealized linear relationship for the average monthly payment per recipient to needy families in the TANF program. Based on this information, what was the average payment to families in 2004?

Average Monthly Payment to TANF Program Recipients



Source: U.S. Department of Health and Human Services.

39. ***Poverty Level Income Cutoffs*** The table lists how poverty level income cutoffs (in dollars) for a family of four have changed over time. Use the midpoint formula to approximate the poverty level cutoff in 2006 to the nearest dollar.

Year	Income (in dollars)
1980	8414
1990	13,359
2000	17,604
2004	19,307
2008	22,025

Source: U.S. Census Bureau.

40. ***Public College Enrollment*** Enrollments in public colleges for recent years are shown in the table. Assuming a linear relationship, estimate the enrollments for **(a)** 2002 and **(b)** 2006. Give answers to the nearest tenth of thousands if applicable.

Year	Enrollment (in thousands)
2000	11,753
2004	12,980
2008	13,972

Source: U.S. Census Bureau.

41. Show that if M is the midpoint of the line segment with endpoints $P(x_1, y_1)$ and $Q(x_2, y_2)$, then $d(P, M) + d(M, Q) = d(P, Q)$ and $d(P, M) = d(M, Q)$.

42. Write the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ using a rational exponent.

For each equation, ***(a)*** *give a table with at least three ordered pairs that are solutions, and* ***(b)*** *graph the equation.* ***See Examples 7 and 8.***

43. $y = \frac{1}{2}x - 2$ **44.** $y = -x + 3$ **45.** $2x + 3y = 5$

46. $3x - 2y = 6$ **47.** $y = x^2$ **48.** $y = x^2 + 2$

49. $y = \sqrt{x - 3}$ **50.** $y = \sqrt{x} - 3$ **51.** $y = |x - 2|$

52. $y = -|x + 4|$ **53.** $y = x^3$ **54.** $y = -x^3$

Concept Check *Answer the following.*

55. If a vertical line is drawn through the point $(4, 3)$, at what point will it intersect the x-axis?

56. If a horizontal line is drawn through the point $(4, 3)$, at what point will it intersect the y-axis?

57. If the point (a, b) is in the second quadrant, in what quadrant is $(a, -b)$? $(-a, b)$? $(-a, -b)$? (b, a)?

58. Show that the points $(-2, 2)$, $(13, 10)$, $(21, -5)$, and $(6, -13)$ are the vertices of a rhombus (all sides equal in length).

59. Are the points $A(1, 1)$, $B(5, 2)$, $C(3, 4)$, and $D(-1, 3)$ the vertices of a parallelogram (opposite sides equal in length)? of a rhombus (all sides equal in length)?

60. Find the coordinates of the points that divide the line segment joining $(4, 5)$ and $(10, 14)$ into three equal parts.

2.2 Circles

- Center-Radius Form
- General Form
- An Application

Center-Radius Form By definition, a **circle** is the set of all points in a plane that lie a given distance from a given point. The given distance is the **radius** of the circle, and the given point is the **center.**

We can find the equation of a circle from its definition by using the distance formula. Suppose that the point (h, k) is the center and the circle has radius r, where $r > 0$. Let (x, y) represent any point on the circle. See **Figure 13.**

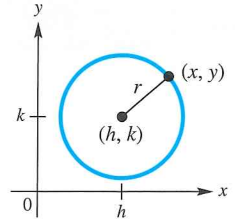


Figure 13

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = r \quad \text{Distance formula (Section 2.1)}$$

$$\sqrt{(x - h)^2 + (y - k)^2} = r \quad (h, k) = (x_1, y_1) \text{ and } (x, y) = (x_2, y_2)$$

$$\mathbf{(x - h)^2 + (y - k)^2 = r^2} \quad \text{Square each side. (Section 1.6)}$$